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Corrigé
Examen en Mécanique des fluides 3

Exercice N°1 06 pts

Les composantes de l'accélération en Coordonnées Cartésiens :

$$\Gamma_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \omega\frac{\partial u}{\partial z} = 0+(2xy)2y+(4tz^2)2x+(-yz)0 = 4xy^2 + 8txz^2$$

$$\Gamma_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \omega\frac{\partial v}{\partial z} = 4z^2+0+0+(-yz)8tz = 4z^2 - 8tyz^2$$

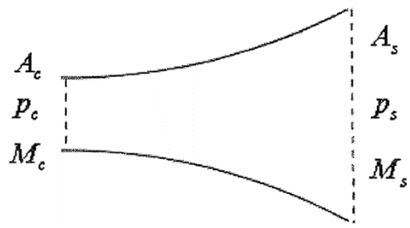
$$\Gamma_z = \frac{\partial \omega}{\partial t} + u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y} + \omega\frac{\partial \omega}{\partial z} = 0+0+(-yz)z-(-yz)y = -yz^2 - zy^2$$

$$\Gamma = \Gamma_x\vec{i} + \Gamma_y\vec{j} + \Gamma_z\vec{k} = (4xy^2 + 8txz^2)\vec{\imath} + (4z^2 - 8tyz^2)\vec{\jmath} - (yz^2 + zy^2)\vec{k}$$

L'accélération particulaire au point (2,-1,1) à t=2s ;

$$\Gamma = (4xy^2 + 8txz^2)\vec{\imath} + (4z^2 - 8tyz^2)\vec{\jmath} - (yz^2 + zy^2)\vec{k} = 40\vec{\imath} - 12\vec{\jmath} + 0\vec{k}$$

Exercice N°2 07 pts



- Le nombre de Mach à la sortie :

puisque $1 + \frac{\gamma-1}{2}M_S^2 = \left(\frac{P_0}{P_S}\right)^{\frac{\gamma-1}{\gamma}}$ afin d'éliminer le P_0 on introduit le rapport à l'entrée $1 + \frac{\gamma-1}{2}M_e^2 = \left(\frac{P_0}{P_e}\right)^{\frac{\gamma-1}{\gamma}}$

d'où $\left(\frac{p_s}{p_e}\right)^{\frac{\gamma-1}{\gamma}} = \frac{1+\frac{\gamma-1}{2}M_e^2}{1+\frac{\gamma-1}{2}M_s^2}$;

$$(0.3)^{\frac{0.4}{1.4}} = \frac{1+0.2M_e^2}{1+0.2M_s^2} = 0.709$$

$$1+0.2M_e^2 = 0.709 + 1.418M_s^2$$

$$M_s^2 = 3{,}465$$

$$M_s = 1.86$$

$(0.3)^{\frac{0.4}{1.4}} = \frac{1+0.2M_e^2}{1+0.2M_s^2} = 0.709$ avec $M_e = 2.56$; Par ailleurs $M_s = 1.86$

- Le rapport entre la section de sortie et celle d'entrée :

A l'entrée $\frac{A_e}{A^*} = \frac{1}{M_e}\left(\frac{1+\frac{\gamma-1}{2}M_e^2}{\frac{\gamma+1}{2}}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$ (1)

Et à la sortie $\frac{A_s}{A^*} = \frac{1}{M_s}\left(\frac{1+\frac{\gamma-1}{2}M_s^2}{\frac{\gamma+1}{2}}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$ (2)

Le rapport entre (1) et (2) donne : $\frac{\frac{A_s}{A^*}}{\frac{A_e}{A^*}} = \frac{\frac{1}{M_s}\left(\frac{1+\frac{\gamma-1}{2}M_s^2}{\frac{\gamma+1}{2}}\right)^{\frac{\gamma+1}{2(\gamma-1)}}}{\frac{1}{M_e}\left(\frac{1+\frac{\gamma-1}{2}M_e^2}{\frac{\gamma+1}{2}}\right)^{\frac{\gamma+1}{2(\gamma-1)}}} = \frac{A_s}{A_e}$ (3)

Après calcul on trouve que $\frac{A_s}{A_e}$=0.461

Exercice N°3 07 pts

Remarque : Il faut prendre le nombre de Mach du débit de gaz à l'entrée du conduit.

$$\int_{p_e}^{p_s} \frac{\mathrm{dp}}{\mathrm{p}} = \gamma \mathrm{M}_e^2 . \frac{1+(\gamma-1)\mathrm{M}_e^2}{2(1-\mathrm{M}_e^2)} \frac{f}{D} \int_0^l dx$$

$$ln\left(\frac{p_s}{p_e}\right) = \left(\gamma \mathrm{M}_e^2 . \frac{1+(\gamma-1)\mathrm{M}_e^2}{2(1-\mathrm{M}_e^2)} \frac{f}{D}\right) . l$$

p_s= 1,054bar

Sachant que : $\frac{dP}{P} = \gamma M^2 \frac{1+(\gamma-1)M^2}{2(1-M^2)} f \frac{dx}{D}$ $\Longrightarrow$ $\int_{pe}^{ps} \frac{dP}{P} = \gamma M^2 \frac{1+(\gamma-1)M^2}{2(1-M^2)} \int_0^l \frac{dx}{D}$

$P_s - P_e = \gamma M^2 \frac{1+(\gamma-1)M^2}{2(1-M^2)}$ $\Longrightarrow$ $\Longrightarrow$

EXAMPLE 3.2 For the velocity field $\mathbf{V} = 2xy\mathbf{i} + 4tz^2\mathbf{j} - yz\mathbf{k}$, find the acceleration, the angular velocity about the z-axis, and the vorticity vector at the point $(2, -1, 1)$ at $t = 2$.

Solution: The acceleration is found as follows:

$$\begin{aligned}\mathbf{a} &= u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y} + w\frac{\partial \mathbf{V}}{\partial z} + \frac{\partial \mathbf{V}}{\partial t} \\ &= 2xy(2y\mathbf{i}) + 4tz^2(2x\mathbf{i} - z\mathbf{k}) - yz(8tz\mathbf{j} - y\mathbf{k}) + 4z^2\mathbf{j}\end{aligned}$$

At the point $(2, -1, 1)$ and $t = 2$ there results

$$\begin{aligned}\mathbf{a} &= 2(2)(-1)(-2\mathbf{i}) + 4(2)(1^2)(4\mathbf{i} - \mathbf{k}) - (-1)(1)(16\mathbf{j} + \mathbf{k}) + 4(1^2)\mathbf{j} \\ &= 8\mathbf{i} + 32\mathbf{i} - 8\mathbf{k} + 16\mathbf{j} + \mathbf{k} + 4\mathbf{j} \\ &= 40\mathbf{i} + 20\mathbf{j} - 7\mathbf{k}\end{aligned}$$

The angular velocity component Ω_z is

$$\Omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}(0 - 2x) = x$$

At the point $(2, -1, 1)$ and $t = 2$ it is $\Omega_z = 2$.

The vorticity vector is

$$\begin{aligned}\boldsymbol{\omega} &= \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)\mathbf{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)\mathbf{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\mathbf{k} \\ &= (-z - 8tz)\mathbf{i} + (0 - 0)\mathbf{j} + (0 - 2x)\mathbf{k}\end{aligned}$$

At the point $(2, -1, 1)$ and $t = 2$ it is

$$\begin{aligned}\boldsymbol{\omega} &= (-1 - 16)\mathbf{i} - 4\mathbf{k} \\ &= -17\mathbf{i} - 4\mathbf{k}\end{aligned}$$

Distance is usually measured in meters and time in seconds. Thus, angular velocity and vorticity would have units of m/(s·m) or rad/s.